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COMBINED STATIC AND DYNAMIC OPTIMIZATION OF A TURBINE DISK

Li Jun, Fan Ning, Zhao Xuecheng
Beijing Power Machinery Institute
Beijing 100191, China

Abstract

Due to high working temperature and rotating speed, turbine disks are crucial parts in gas turbine engines. The weight of disks is always heavy in order to increase the reliability and structural integrity. So optimization design of disks could bring a significant reduction in engine weight. Focusing on a typical Low Pressure Turbine(LPT) disk, this paper improves its design in both static and dynamic characteristics with ANSYS Workbench platform. Based on a 2D parameterized model, the sensitivity of different structural parameters was investigated quickly. Then the optimization process to minimize the mass was conducted by NLPQL (Non-Linear Programming by Quadratic Lagrangian) method with 3D parameterized model. The equivalent stress of disk was limited in static optimization and resonance frequency was also restricted to a safe level through a Campbell diagram in dynamic optimization. A new design plan was acquired through optimization process, which reduces 13.6% of total weight under static and dynamic criteria.

Introduction

The demand of power and fuel cost leads to more efficient designs in turbomachinery, but it makes working condition of engine worse and the structure reliability becomes a serious problem. Turbine disks comprise a large part of the structural weight of an engine, while facing both static and dynamic problems. Methods for safely decreasing the engine weight must be investigated.

Limited by calculation resources, one dimensional stress models for disks of varying thickness have been in use for a long time. Simple models for isotropic disks are available in many sources[1,2,3].The disk design routine in GasTurb Details is the most widely available of these codes[4]. The development and implementation of a low fidelity stress model and rapid optimization procedure for turbomachinery disks has been presented by Gutzwiller[5].Three traditional geometry definition methods are compared to two new methods that are described and produce more optimum designs. However, the literature above mainly focuses on the stress level and the dynamic characteristics of disk could not be considered simultaneously. It may cause re-design work or even safety problem in the future.

In research of optimization methods, Fox[6] optimized a rotating disk using the feasible direction method and Zienkiewicz and Combell[7] used sequential linear programming methods for optimization of rotating disk. Also, in a different approach, Chen[8] optimized a rotating disk profile by combining the finite element method with the feasible direction method. Jafari[9] finding an optimal disk profiles for minimum weight design using the Karush-Kuhn-Tucker method (KKT) as a classical optimization method, simulated annealing (SA) and particle swarm optimization (PSO) as two modern optimization techniques.

Focusing on a typical Low Pressure Turbine(LPT) disk, this paper improves its design in both static and dynamic characteristics with ANSYS Workbench platform. An optimization model was illustrated accordingly. The sensitivity of different structural parameters was investigated by using a simple 2D parameterized model.

Then the optimization process to minimize the mass was conducted by NLPQL (Non-Linear Programming by Quadratic Lagrangian) method. The equivalent stress of disk was limited and resonance frequency was also restricted to a safe level through a Campbell diagram analysis. Finally, a new design plan was acquired through optimization process, which reduces 13.6% of total weight under static and dynamic criteria.

1. Optimization Model

1.1 Optimization process

The whole optimization process which combines static and dynamic optimization is shown in Fig.1. First of all, a parameterized model needs to be built, which means that the structure should be fully defined by size parameters. The mesh of Finite Element (FE) model could be automatically generated when those parameters change. Then a sensitivity analysis will conducted in order to find the most critical parameters to stress result and structure total mass. Since there are always a large number of parameters in describing a complex geometry, searching the sensitive size could take quite long time. So it is recommended to use 2D plane stress model in this step and choose parameters according to the experience of mechanics.

After the sensitivity analysis, a static optimization result can be obtained, which will be transferred to dynamic optimization. Finally, the program will calculate the resonance speed of blisk by Campbell diagram under dynamic requirement. Although using 2D plane model would be more efficient in stress analysis, the combined optimization still uses 3D model, which is more compatible for dynamic analysis. After optimization, the final geometry could meet both static and dynamic criteria.

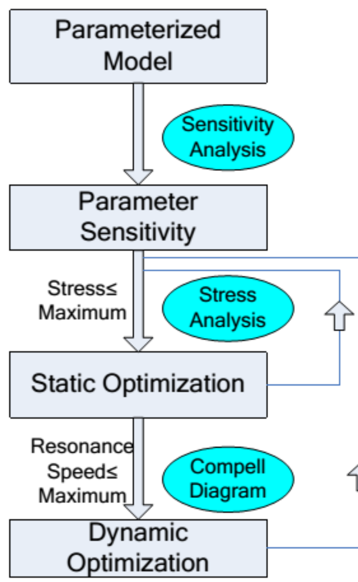


Fig. 1 Optimization process

1.2 Mathematical model

As a form of single-objective problem, the mathematical model of it can be expressed as:

find: $\boldsymbol{X}$ at $[\boldsymbol{X}_L, \boldsymbol{X}_U]$

minimize: $f(\boldsymbol{X})$

subject to: $h_i(\boldsymbol{X}) \le \sigma_{max}$ $i=1,2\ldots m$ (1)

$g_i(\boldsymbol{X}) \le \omega_{min}$ and $g_i(\boldsymbol{X}) \ge \omega_{max}$ $j=1,2\ldots n$

where $\boldsymbol{X}=\{X_1,X_2 \ldots X_r\}$ is the vector of design parameters. Due to structural limitation, $\boldsymbol{X}$ has its low boundary $\boldsymbol{X}_L$ and up boundary $\boldsymbol{X}_U$. $f(\boldsymbol{X})$ is the objective function. In this case, total mass of disk is the only objective. $h_i(\boldsymbol{X})$ represents stress of critical positions, which is constrained under σ_{max}. All of resonance speed points $g_i(\boldsymbol{X})$ should be kept away from working speed range of rotor, in order to avoid resonance.

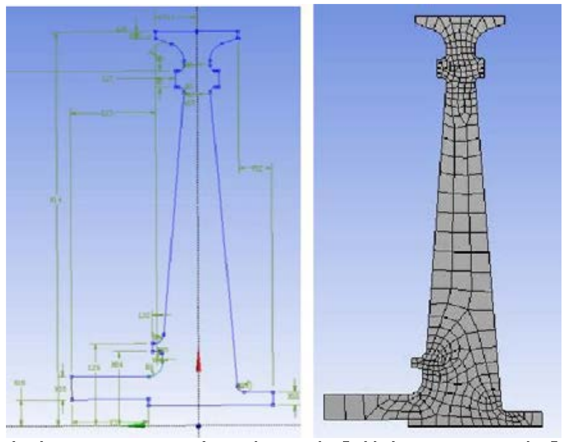
1.3 NLPQL method

The Non-linear Programming by Quadratic Lagrangian (NLPQL) method is used in the optimization process. It is assumed that objective function and constraints are continuously differentiable. The idea is to generate a sequence of quadratic programming subproblems obtained by a quadratic approximation of the Lagrangian function and a linearization of the constraints. Second order information is updated by a quasi-Newton formula and the method is stabilized by an additional (Armijo) line search.

2. Parameter Sensitivity Analysis

2.1 2D parameterized model

As shown in Fig. 2, the low pressure turbine (LPT) disk was parameterized into 27 dimensions in Design Modeler module in ANSYS Workbench. Based on real structure, the details of geometry had been reserved as much as possible. A 2D plane stress model with axisymmetric assumption was built accordingly. In this chapter, the geometry of blade was simplified into centrifugal force on outer rim.



(a)Parameterized model(b) FE model
Fig. 2 Model for sensitivity analysis

The radius and width of outer rim are defined by the blade and blade root geometry, which is usually held static throughout optimization process. Due to structural limitation, the bore radius and some other parameters were also kept constant in this case. After changing the all remaining dimensions, two sensitive parameters were found, shown in Fig. 3. The result reveals that the total mass, objective function $f(\boldsymbol{X})$, is influenced obviously by the width and angle of web.

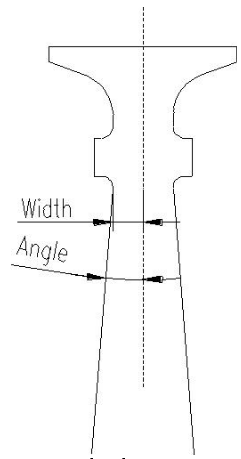


Fig. 3 Sensitive parameters

2.2 Sensitivity analysis

The sensitivity of these two parameters on equivalent stress of bore and total mass were compared in Fig. 4. On one hand, the total mass is in positive correlation with both them, while the mass is much more sensitive with web width than angle. On the other hand, the equivalent stress of bore, whose radius is the smallest in disk and always becomes the most fragile part, will increase significantly if the web becomes thinner. In addition, the angle affects the stress more than width. The response charts of stress and mass with the change of width and angle in possible range are shown in Fig. 5. A preliminary conclusion can be found that the optimization should focus on two dimensions of web and the total mass can be reduced more by decreasing web width with little cost of stress level.

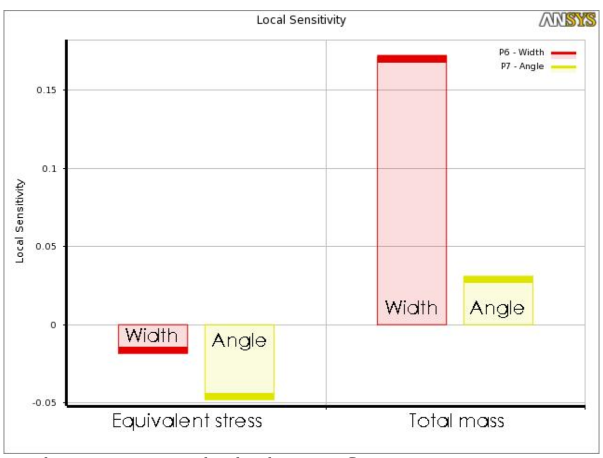
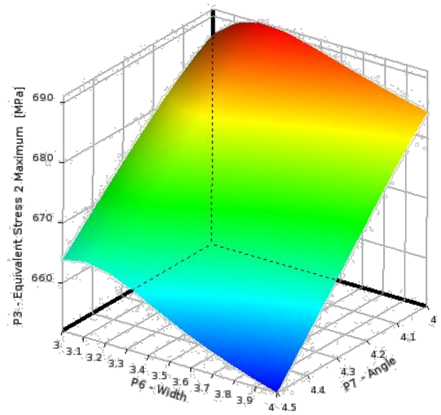
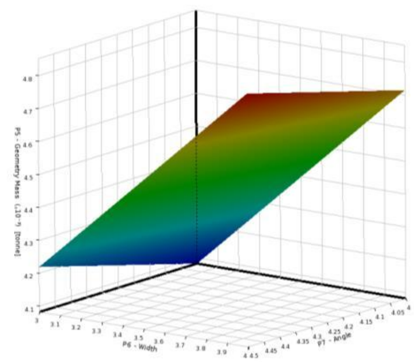


Fig. 4 Sensitivity of parameters on stress and mass



(a) Stress response



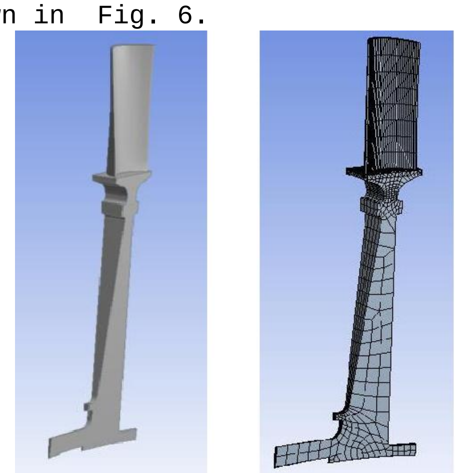
(b) mass response

Fig. 5 Response with parameters

<u>3. Static & Dynamic Optimization</u>

3.1 3D parameterized model

Based on 2D parameterized model, a 3D parameterized model was developed, shown in Fig. 6.



(a) Parameterized model (b) FE model

Fig.6 3D model for optimization

Since there are 69 blades in the blisk, the parameterized plane was swept into a 1/69 section. The blade geometry was added on the outer rim of disk and its elements were connected with disk by a few of tetrahedral elements. With cyclic symmetry assumption, the FE model contains 6518 nodes and 6522 elements.

3.2 Combined optimization

Considering the result of parameter sensitivity, there are two critical parameters- width X_1 and angle X_2 in combined optimization. So:

$$\boldsymbol{X}=\{X_1,X_2\}, \tag{2}$$

while $X_1\in[3,4]$, $X_2\in[4.4.5]$.

In static optimization, the von Mises stress of the disk should be less than elastic limit. A safety factor was introduced in the design, making the maximum von Mises stress of optimum disk σ_e as 0.85 times of yield strength of the material σ_s. So the static criteria could be expressed as follow:

$$\sigma_e \le 0.85\sigma_s \tag{3}$$

In dynamic optimization, all resonance points $g_i(\boldsymbol{X})$ should be kept away from working speed range. In this case, the rotor rotates in constant speed ω_c. A 20% margin of resonance was assumed, so the dynamic criteria is:

$$g_i(\boldsymbol{X}) \le 0.8\omega_c \text{ and } g_i(\boldsymbol{X}) \ge 1.2\omega_c \quad j=1,2\ldots n \tag{4}$$

which can be also expressed :

$$\eta \ge 0.2 \ , j=1,2\ldots n \tag{5}$$

where η represents the margin of resonance.

Table 1 shows the initial geometry and material properties used in the optimization.

Table 1 Geometry and material properties

X_1/mm	X_2/°	E/GPa	σ_s/MPa	ω_c/rpm
4	4.5	181	788	21000

Using optimization model discussed before, a combined static and dynamic optimization was conducted by NLPQL method. The design parameters before and after optimization are shown in Table 2

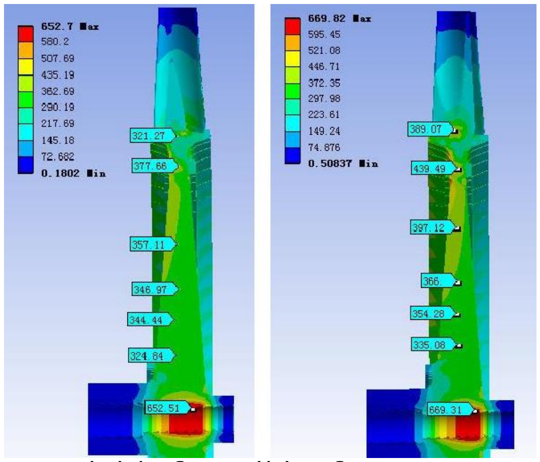
Table 2 Optimization results

	X_1/mm	X_2/°	σ_e/MPa	η	m/kg
before	4.0	4.5	653	0.36	4.85
after	3.0	4.38	670	0.27	4.19

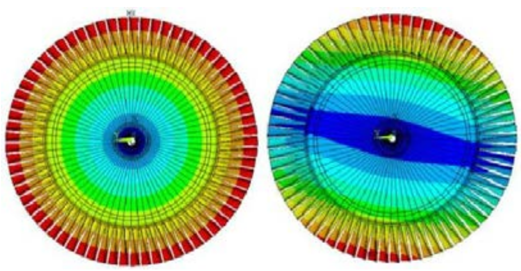
The results reveal that X_1(width of web) has reached its boundary. It contributes more than X_2(angle of web) to the objective function $f(\boldsymbol{X})$ (mass), which coincides with the conclusion of sensitivity analysis. Due to the limitation of stress level, X_2 becomes to 4.38°. It should be also noticed that the resonance margin η has no reached the limitation. As a result, this design has a weight reduction of 13.6%.

3.3 Stress analysis

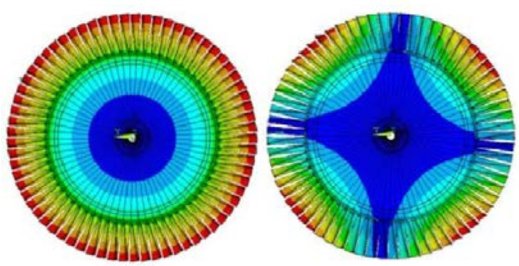
Stress plots of blisk before and after optimization are shown in Fig. 7. It can be inferred from both of structure that the maximum value of von Mises stress σ_e occurs at the bore, while the stress level of web is relatively low.



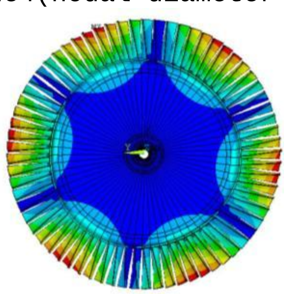
(a) before (b) after
Fig. 7 Von Mise stress in disk



(a)Mode1(torsional) (b) Mode2(nodal diameter =1)



(c) Mode3(nodal circle) (d) Mode4(nodal diameter=2)



(e) Mode5(nodal diameter=3)
Fig. 8 First 5 modes of blisk

Although σ_e of optimized disk reached $0.85\sigma_s$, stress at its web region still remains in range of 300~450MPa, which represents enough stress margin to the yield strength of material.

3.4 Dynamic analysis

Before dynamic optimization, the modal characteristics of blisk were evaluated. As shown in Fig. 8, first 5 modes were calculated. Among these modes, only nodal diameter modes can be excited in rotating, so the torsional direction mode and nodal circle mode were excluded deliberately in optimization.

Then a Campbell diagram analysis was performed in dynamic analysis. The frequency of nodal diameter modes under every design point was calculated and the resonance speed under 1X~3X rotating frequency excitation was conferred. Although some modes could not be excited easily by three kinds of excitation force, all possible resonance speed was compared with the working speed ω_c.

The Campbell diagram is shown in Fig. 9. The dash line represents the frequency of original blisk and the solid line plots the optimized one. *m* or *m′* is the nodal diameter number of each mode. Obviously, the frequency of optimized blisk becomes lower than before. The nearest resonance speed of optimized blisk to working point is 26400rpm, which remains 25.7% resonance margin.

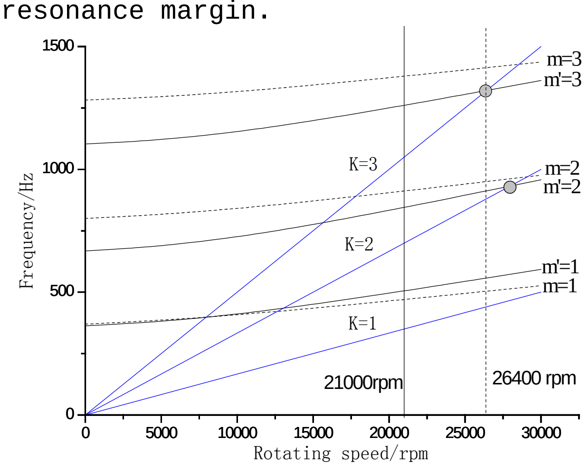


Fig. 9 Campbell diagram

4. Conclusions

An optimization model combines static and dynamic optimization for disk has been presented in this paper. The optimization process includes four parts: building parameterized model, sensitivity analysis, static optimization and dynamic optimization. A 2D parameterized plane stress model was used in sensitivity analysis due to its simplicity and speed. Based on sensitivity analysis, the optimization process can find the most sensitive parameters deciding the objective function and stress level. Then a 3D parameterized model was built in order to get the dynamic characteristics of blisk. By using NLPQL method, the optimization would minimize the weight of disk in constrain of stress and dynamic criteria.

For the LPT blisk discussed, sensitivity analysis discovered two critical parameters among all changeable parameters: width and angle of the web. By optimizing these parameters, the design reduces 13.6% of total mass with a little cost of stress increase at the bore and the resonance margin to working speed is still large.

In the future, the design plan may apply on an experimental disk. And further analysis including thermal-structural deformation, plasticity stress, low cycle failure (LCF) life should be conducted carefully.

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